Consciousness in Enacting Procedural Knowledge

John Selden and Annie Selden
Department of Mathematical Sciences
New Mexico State University

Abstract. We describe a perspective for examining the enactment of a common kind of procedural knowledge and how that enactment relates to consciousness. Here, we view procedural knowledge in a very fine-grained way, for example, considering a single step in a procedure, and discuss knowledge that includes, not only how to, but also to, or when to, physically or mentally act. We call the mental structure that links information allowing one to recognize that an act is to be performed, to what is to be done and how to do it, a *behavioral schema*. We consider how such behavioral schemas might be enacted and how they might interact. The processes associated with a schema's enaction appear to occur outside of consciousness, but some information triggering its enaction is conscious, and the resulting action is conscious or immediately becomes conscious. We include examples as simple as calculating (10/5) + 7 and mention some implications of this perspective.

1. Introduction

For some time now the mathematics education research literature has had much more to say about topics related to conceptual knowledge than about those related to procedural knowledge. This can be seen, for example, reflected in recent papers in the Educational Resources Information Center (ERIC) and the Journal for Research in Mathematics Education (*JRME*) (Star, 2005, p. 405).

One might also expect that in the construction of a proof, a well-developed form of conceptual reasoning, that the role of conceptual knowledge would dominate that of procedural knowledge. In particular, one might expect that one's actions in the proving process were caused, in some immediate sense, by one's conceptual reasoning. However, as far as we can see, when we construct proofs we "just do" many of the actions. That is, we are often not conscious of any causal conceptual reasoning just prior to an action, even though we could have later provided a justification for it.

For example, if the statement of a theorem starts, explicitly or implicitly, with "For all x in the real numbers," then near the beginning of the proof we might write "Let x be a real number," meaning that x is to represent, not a variable, but a particular arbitrary, that is, unspecified, real number. For such actions in writing proofs, we have asked ourselves of what were we aware just prior to the action of writing? Apart from the situation calling for the action, that is, that we were proving a theorem whose statement started with a universally quantified real variable, the answer is *nothing*. In particular, we do not recall being aware of, that is, being conscious of, any kind of reasoning mechanism calling on conceptual knowledge. We could easily have provided such conceptual reasoning, for example in inner speech, but did not do so. However, since our actions were not capricious, *something* must have guided them.

What we have described above appears to be an example of the enactment of a kind of procedural knowledge. That knowledge probably arose inductively, when conceptual reasoning or understanding indicated that a particular kind of situation called for a particular kind of action and the action was carried out – the entire process being repeated for a number of proofs. We suspect that the use of this kind of procedural knowledge to guide action is beneficial in that it places much less burden on working memory than would the corresponding conceptual knowledge-based reasoning justifying the action. Furthermore, for most people the use of this kind of procedural knowledge seems to be quite common and may include triggering or guiding mental actions such as searching for a particular piece of conceptual knowledge (i.e., trying to remember it), or drawing an inference from recently brought to mind knowledge, such as an inference justified by modus ponens. Thus, much of the use of conceptual knowledge in proving, reasoning, and problem solving may be controlled by the enactment of the kind of small grain sized procedural knowledge discussed above.

In this paper, we initiate an exploration of such procedural knowledge. In doing so, we mention situations to which procedural knowledge responds in directing actions. These situations encode information representing part of the current state of affairs during an ongoing cognitive process and can include feelings, such as a feeling of rightness, that have mainly cognitive, rather than emotional, content. Feelings of this kind have not been widely investigated in the mathematics education research literature and their discussion is beyond the scope of this paper. However, such feelings are discussed in Selden, Selden, and McKee (2008).

In suggesting a perspective and a framework for understanding how the above kind of small grain sized procedural knowledge is used in constructing proofs, we will draw on, and slightly extend, ideas from mathematics education, psychology, neurology, and philosophy. Along the way, we point out a certain mental structure that we call a *behavioral schema*. This will be followed by a few informal observations that provide some modest justification for this perspective and framework. Finally, we suggest a theoretical view of the enactment of behavioral schemas and mention some implications for teaching.

2. Perspective

2.1 Procedural knowledge

We see knowledge in two rather different ways. On the one had, we see it "from the inside" as contained in a person's mind, embodied in neurological structures and processes, and detectable only through the person's communications and actions. On the other hand, we see knowledge "from the outside" as abstract and visible mainly in texts (speech, writing, diagrams, etc.) that pass among persons. In this paper, we will focus on the first view – knowledge as contained in a person's mind.

In the mathematics education community, it is also common to distinguish procedural from conceptual knowledge. This distinction is sometimes described by attending to the nature of the knowledge or how it is to be used. From this perspective, procedural knowledge is seen as *knowing how* to so something, and in contrast, conceptual knowledge is seen as *knowing that, or why*, something is true. For example, knowing the Pythagorean theorem is conceptual knowledge and knowing how to solve quadratic equations is procedural knowledge. Another description, favored by Hiebert and Lefevre (1986) calls for attending to the structure of knowledge, that is, the links between the various units of knowledge. From this perspective, a unit of conceptual knowledge is seen as part of a rich "web" of linking relationships of many kinds. For example, a person may know the Pythagorean theorem provides an explanation of Euclidean distance in the plane, and that such distance satisfies the triangle inequality, which generalizes to part of the definition of metric space, which in turn, yields examples of Hausdorff topological spaces. In contrast, procedural knowledge is seen as consisting of knowledge of the symbols and syntax of mathematics, together with rules, algorithms, or procedures – in essence step-by-step instructions. The relationships consist mainly just in going to the next step in a predetermined and largely linear way, although some branching may be required, for example, in subtracting multi-digit numbers. Also, we note that the step-by-step instructions may be in the eye of the beholder. For example, a person riding a bicycle might be seen as exhibiting procedural knowledge, but is likely not to be aware of following any step-by-step instructions.

Starting with the above (structural) descriptions favored by Hiebert and Lefevre (1986), Star (2005) has pointed out that procedural knowledge is not necessarily superficial. He suggests a dimension of quality, ranging from superficial to deep, should be added to both procedural and conceptual knowledge. While we agree that it can often be very useful to attend to both the structure (e.g., connectedness) and the quality of knowledge, in this paper we are concerned with how mathematics is done, for example, how proofs are constructed, in a moment-to-moment way. With this in mind, we focus on the first, and perhaps older, of the above descriptions, attending to the nature of knowledge or how it is to be used, and take procedural knowledge to be *knowing how* to do something. This allows procedural knowledge to include knowing how to wrie small parts of proofs that are not parts of a procedure.

Procedural knowledge can be displayed in two ways: (1) a person might describe how to do something, or (2) be able to actually carry it out. These are not equivalent – a person may be able to describe how to do something, but not actually be able to do it, and vice versa. For example, there are students who can recite how to row reduce a matrix, but cannot often readily do so. Also, there are surely persons able to do long division that would have difficulty articulating exactly how it is done. Indeed, a person might not even be aware of having certain procedural knowledge. For example, in attempting to prove that every differentiable function on the real numbers is continuous, a person might start with “Let f be a function and suppose f is differentiable,” without needing to recall anything about the ways proofs are written and without thinking of this as part of knowing how to write a proof. Such knowledge is reminiscent, in the conceptual realm, of theorems-in-action (Vergnaud, 1990). Here, we will mean by procedural knowledge the ability to actually carry out an action, whether or not the person can articulate how it is done.

2.2 Grain-size

In addition to taking procedural knowledge to be the ability to actually carry out an action, we include actions of a very small grain-size. Thus, if the action is a procedure of several steps, we will also focus on single steps in the procedure. We take such steps in themselves, to be procedural knowledge. For example, in solving $2x + 5 = x + 3$, the ability to "remove the 5 from the left side and put -5 on the other side" will be taken as procedural knowledge. This can be regarded as a step in a larger procedure, the ability to solve (certain kinds of) linear equations. However, we also include in procedural knowledge, small abilities for which no such containing procedure is apparent. For example, consider constructing the proof of a theorem which says, or means: For all real numbers x, if $P(x)$ then $Q(x)$. For students familiar with constructing proofs, it will normally be part of procedural knowledge to start the proof with something that means: *Let x be a real number. Suppose P(x).* This can be said in several ways, not only as we have said it, but also by excluding any mention of *Let x be a real number*, in which case its meaning should nevertheless be understood by both the reader and writer. That is, the x will be treated as representing a particular, but unspecified, real number, rather than as representing the variable in the statement of the theorem. It would be hard to see the above writing of the first part of a proof as part of a general, larger "theorem proving procedure" and to identify a specific "next" step.

Seeing procedural knowledge in this small-grained way suggests it occurs very widely throughout mathematics. For example, in situations involving a function *f* from *X* to *Y* and a subset *A* of *Y*, beginning mathematics graduate students will know that $f^{-1}(A)$ is defined (conceptually) to be $\{x| f(x) \in A\}$. However, in the context of constructing a proof, for many students this will correspond to the procedural knowledge that, given $b \in f^{-1}(A)$, it is legitimate to claim $f(b) \in A$, and vice versa. We have had students who could state the concept definition, but in practice did not exhibit the corresponding procedural knowledge by carrying out the indicated action in the appropriate situation.

2.3 Actions

In discussing a person's procedural knowledge as knowing how to do things, meaning the person has the ability to actually do them, we are referring to the person's ability to act. But actions can be physical or mental. In this paper, we will not distinguish mental actions from physical actions. This seems to be in agreement with most previous uses of procedural knowledge, and in what we are discussing, the distinction is largely immaterial. This is often the case in mathematics and mathematics education. For example, consider a person solving $3x + 5 = 17$. From the point of view of mathematics, what matters is that the answer is 4, as opposed to say, 5, and not whether the 4 occurred physically, say written on a paper, or mentally, say in inner speech. Of course, there are other contexts where the physical-mental distinction does matter. For example, one might wish to point out that in problem solving the specifically physical act of writing can reduce the burden on a person' working memory.

Margolis (1993) also found it useful to purposely conflate physical and mental acts when using habits of mind to explain how difficult it is for individuals to participate in scientific revolutions. However, in future research, some caution with regard to mental versus physical acts may be called for. One might expect that procedural knowledge (as understood by the mathematics education community) might be encoded in procedural memory (as understood in the memory literature in psychology). But procedural memory refers to *physical* acts (Tulvig, 2000, p. 37), perhaps because many psychological experiments involve nonhuman animals, who of course cannot report mental acts. Calling on the memory literature may involve inferring ideas about mental acts from results about physical acts.

3. A framework

3.1 Behavioral schemas

We adjoin to procedural knowledge (i.e., knowing *how* to act) what Mason and Spence (1999) have called "knowing *to* act in the moment" to obtain enduring mental structures that allow a person to connect a situation with an action. We refer to those enduring mental structures as *behavioral schemas* and see them as a part of a person's knowledge base. The action might be simple or complex, such as a procedure consisting of several smaller actions each produced by the enactment of its own behavioral schema.

Although we are suggesting behavioral schemas as enduring mental structures, we are not claiming anything about properties of their neurological embodiment. This is similar to Tall and Vinner's (1981) introduction of concept images as mental structures without a description of their neurological embodiment. However, we suggest ways behavioral schemas and their enactments relate to memory, learning, consciousness, and the enactment of other behavioral schemas.

A behavioral schema need not *correctly* link a situation to an action. For example, a behavioral schema might cause a student to regularly make a cancellation error such as simplifying "$(2a+3)/2b$" [the situation] by writing "$(a+3)/b$" [the action].

That a behavioral schema can produce an action that goes beyond simple mathematical manipulation is illustrated in Weber (2008, Section 4.2.2). He reports that he asked students to prove a theorem after reading a text including a definition, examples, and several proofs. Some of his less successful mathematics majors searched for, and found, a theorem proved in the text that they thought was similar. They then simplistically mimicked the proof of the earlier theorem even though, on later questioning, they could see their resulting argument was not correct, and indeed could produce a correct proof themselves. Weber's description suggests to us that his less successful mathematics majors had a behavioral schema that rather unfortunately linked a request for a proof in the presence of earlier proofs [the situation] to simplistically mimicking a proof and not checking the result [the action].

3.2 Situations

By a situation, as used above, we mean a part of the information available to a person at any one time, and we are treating situations as dynamic (i.e., changeable) and ephemeral. Also we are referring to a person's inner, interpreted situation, not just the external or immediately perceived situation, for example something that can be written.

Norton and D'Ambrosio (2008) provide us an illustration of this later distinction for two middle school students, Will and Hillary, who view the same external situation involving a fraction such as 2/3. Hillary had (in her knowledge base) a *partitive* *fractional scheme*, as well as a *part-whole fractional scheme*, while Will had only the second scheme. This caused Will and Hillary to "see" the external situation differently, that is, to have differing inner situations, and hence, act differently. In particular, Hillary was able to solve a problem that Will could not. Will could solve the problem only after he had developed a partitive fractional scheme, and presumably then experienced a richer inner situation.

In the above illustration, Will's internal view of the external situation could not be enriched by a concept of fraction that was not yet available in his knowledge base. But that is not the only way for two persons to have significantly differing inner situations for the same external situation. Selden, Selden, Hauk and Mason (2000) reported on mid-level undergraduates in a first course on differential equations attempting to solve moderately non-routine beginning calculus problems. The students did not know how to solve the problems, but they called on calculus facts that the students were familiar with, and the problems were also somewhat similar those that the students could have been expected to know how to solve. For example, one problem asked: Does $x^{21} + x^{19} - x^{-1} + 2 = 0$ have any roots between –1 and 0? Why or why not? This problem could not be solved by simple algebraic techniques, but could be solved by appealing to the Intermediate Value Theorem, and calculating the corresponding function's maximal value over [-1,0]. Selden, et al. were able to show that a number of the students could not solve moderately non-routine problems for which they had adequate information in their knowledge bases. Apparently, the students were unable to bring this information to mind, that is, into consciousness, because their knowledge bases lacked links between "kinds" of problems and information that might be useful in solving them. Thus, the students were unable to enrich their views of the external situations to create inner situations (including connections to things like the Intermediate Value Theorem) that might have stimulated the enactment of appropriate problem-solving behavioral schemas.

4. Consciousness

4.1 Phenomenal consciousness

In the following section, we need to refer to consciousness. In particular we discuss some mental states or processes, that are, or are not, conscious.

Consciousness has been discussed from several perspectives including the perspective of the self (that only a few nonhuman animals seem to have to any significant degree) and of altered states of consciousness (e.g., those that occur during some kinds of meditation). These perspectives are beyond the scope of this paper. Here we refer only to phenomenological consciousness, that is, just to the subjective experiences that everyone seems to be aware of in various situations. For example, this includes experiences arising from the senses (vision, hearing, etc.), one's own speech, and other physical actions, as well as the corresponding inner versions (inner vision, inner speech, etc.). In addition, consciousness can include more subtle experiences, often without a sensory component, such as feelings (e.g., the feeling that a proof is correct) and intentions (e.g., the intention to first combine the x terms in solving a linear equation).

Also at any one time, one can focus upon part of what one is conscious of, without completely neglecting the rest. And this focus can be moved around within one's conscious field, partly guided by the portion not yet focused upon (i.e., what James (1890) might call the "fringe"). This is rather like vision consisting of a central, high resolution part, and a peripheral, low resolution part. The changing of a person's focus probably contributes to the dynamic, ephemeral nature of inner situations.

4.2 Does consciousness drive action?

Most people believe that their everyday actions, such as raising a hand, are caused, in an immediate sense, by their conscious intentions. However, Libet, Gleason, Wright and Pearl (1983) challenged this with an ingenious experiment. The research concerned a freely voluntary, fully endogenous motor act, raising a finger or hand. Through the use of a specially designed clock, subjects reported the times of their intentions to act. In addition, times were recorded for the actual acts, and for a neurological feature that occurs near the beginning of a physical act, the onset of the readiness-potential. It was found that the readiness-potential sometimes precedes consciousness of the intention to act.

Lau, Rogers, Haggard, and Passingham (2004) replicated and strengthened the work of Libet et al. using neurological (e.g., *f* MRI) methods that did not depend on subjects' reports of such short times (~200-300 ms.). Thus, the belief that conscious intent drives actions (in some immediate sense) may be an illusion. Indeed, there has recently been considerable discussion of whether consciousness causes behavior (Pockett, Banks, & Gallagher, 2006), and even whether consciousness might be an epiphenomenon (Pauen, Staudacher, & Walter, 2006).

5. Informal Observations

5.1 An algebraic manipulation

While administering a calculus test, the first author watched a student rewrite a polynomial three times as part of solving a problem. The polynomial ended in “+7”, but the student’s horizontal bar on the “+” sign was only half complete. In the next copy of the polynomial, it ended with “-17”. The error was then propagated through the rest of the solution.

The error tells us very little about the student’s understanding of calculus, but can reveal something of the details of the student’s cognition. It seems very unlikely that anyone has an inner, mental representation for “half a plus sign.” Thus the student could not have made this error mentally and merely recorded it in writing. But this means the student’s writing and then reading of the “-17” was necessary to the student’s writing, or thinking of, the next line of her solution.

Our interpretation of this is that the student became conscious of the polynomial, including the “-17” [the situation], enacted a behavioral schema for copying, wrote the subsequent polynomial on the next line [the action], and became conscious of that polynomial while it was being written or immediately after it was written. The first “-17” was a conscious interpretation (during reading) of the external situation and did not actually exist on the paper, so reading (and also consciousness) was a necessary part of the enactment of the student’s behavioral schema.

5.2 An arithmetic calculation

We have asked several undergraduate and graduate students and a professor to think aloud while calculating (10/5) + 7. In some cases, this was presented in written form, and in others, was read aloud. In all cases, 2 was mentioned before the answer 9 was obtained. In one of these, the 2 was a verbal report of an inner vision of the 2.

Our interpretation of these informal observations is that the task was always decomposed into the enactment of two behavioral schemas, one to obtain 2 from 10/5 and the other to obtain 9 from 2 + 7. The result of the action of the first behavioral schema, that is, the 2, did not combine with the 7 to allow the enactment of the second behavioral schema without first becoming conscious. We suggest that the chaining of the enactments of behavioral schemas outside of consciousness may not be possible.

6. The role of consciousness

6.1 The necessity of consciousness in reasoning

The two informal observations in Section 5 suggest that for the enactment of a behavioral schema to link a situation to an action, the situation must be conscious.

Also in Subsection 3.2, we mention Selden, et al. (2000) indicating that some mid-level undergraduates could not solve certain moderately non-routine problems while having adequate information in their knowledge bases. Apparently to be useful such information must be brought to mind, that is, brought into consciousness, in order to become part of a situation which might then stimulate the enactment of a behavioral schema and lead to an action, and perhaps ultimately, the solution of the problem.

6.2 Processes that occur outside of consciousness

In Section 1, we noted that in our own proving of theorems, our actions do not seem to be immediately preceded by anything conscious. Also in the observations described in Subsection 5.2, in thinking aloud while calculating (10/5) + 7, no one reported being conscious of anything immediately prior to obtaining the 2 from 10/5 or the 9 from 2 + 7. Thus we suggest that whatever mental and neurological processes are involved in the enactment of a behavioral schema, they take place outside of consciousness.

7. A theoretical view of the enactment of behavioral schema.

The above ideas that behavioral schemas act outside of consciousness to guide action and that behavioral schemas depend on the consciousness of a situation to yield an action, can be extended somewhat to suggest the following six-point theoretical sketch of the enactment and origin of behavioral schemas.

1. Within very broad contextual considerations, behavioral schemas are immediately available. They do not normally have to be remembered, that is, searched for and brought to mind.
2. Behavioral schemas operate outside of consciousness. One is not aware of doing anything immediately prior to the resulting action.
3. Behavioral schemas tend to produce immediate action, which may lead to later action. One becomes conscious of the action resulting from a behavioral schema as it occurs or immediately after it occurs.
4. A behavioral schema that would produce a particular action cannot pass that information, outside of consciousness, to be acted on by another behavioral schema. The first action must actually take place and become conscious in order to become information acted on by the second behavioral schema. That is, one cannot "chain together" behavioral schemas in a way that functions entirely outside of consciousness and produces consciousness of only the action of the last behavioral schema. For example, if the solution to a linear equation would

normally require several steps, one cannot give the final answer without being conscious of some of the intermediate steps.

5. An action due to a behavioral schema depends on conscious input, at least in large part. A stimulus need not become conscious to *influence* a person's actions, but such influence is normally not precise enough to trigger a behavioral schema.
6. Behavioral schemas are acquired (learned) through habituation. That is, to acquire such a schema a person should carry out the appropriate action correctly a number of times. Changing an incorrect behavioral schema requires similar, perhaps longer, practice.

The theoretical view of the enactment of behavioral schemas sketched above is a description of the way procedural knowledge connects situations to actions from the inside, or first person, perspective. A very similar phenomenon can also be described from the outside, or third person, perspective. In that case, instead of discussing mental structures and what is, or is not, conscious, one describes habits of mind. Margolis (1993) has developed such a description of habits of mind, taking a large-grained focus in discussing scientific beliefs, rather than mathematical cognition.

8. Teaching

The theoretical framework and perspective sketched in this paper may prove helpful in understanding some difficulties students have in doing mathematics and in suggesting ways to alleviate those difficulties. A hint of this cane be seen in Section 6 of

Selden, Selden, and McKee (2008) where we mention a student's feeling of confusion that leads to the enactment of what might be called a "grasping at straws" behavioral schema. Below we describe two other difficulties students have.

8.1 Counterexamples are not enough

Some experienced teachers may have noticed that giving a counterexample to a student who consistently makes an errorful calculation, sometimes referred to as a "buggy algorithm," like $(3a+b)/3c=(a+b)/c$ or $\sqrt{a^2+b^2}=a+b$, is often not very effective. This can be so even when the student seems to understand the counterexample. This theoretical perspective suggests an explanation. If an incorrect algebraic simplification is caused by the enactment of a behavioral schema occurring outside of consciousness, then the resulting action, the incorrect simplification, would not be under conscious control. To change the student's behavior, one might try to change the offending behavioral schema by not only providing a counterexample, but also a number of relevant problems and some monitoring. By influencing the student to correctly carry out the simplification and manipulations involving this point a number of times, the student might be helped to weaken the old behavioral schema and create a better one. Here, we are not suggesting that the conceptual understanding, perhaps starting with a counterexample, is not valuable to the student, but only that it may be inadequate to change the student's behavior. Indeed, we suspect this might be so, even in rare cases where the student, rather than the teacher, finds the counterexample.

8.2 A difficulty with sets

Maria and José were students in a class meant to help beginning graduate and advanced undergraduate mathematics students improve their proving skills. It was midsemester and the students knew the definition of a topology, had had some experience with theorems about sets, and knew that a closed set is the complement of an open set, but did not yet know about limit points. Maria had written on the blackboard a proof that in a topological space X, any set A is contained in a smallest closed set. José was validating (i.e., reading and checking) her proof out loud. In the proof, it was claimed that $\mathcal{C} = \{C \mid A \subseteq C, C \text{ closed}\}$ is nonempty because $X \in \mathcal{C}$. José had difficulty seeing this and asked Maria why X is an element of $\mathcal{C}$. Maria replied that X is closed because it is the complement of the empty set which is open (a response that answered a different question that José did not ask). José still did not understand. It turns out that José did not see that he could show $X \in \mathcal{C}$ by showing $A \subseteq X$ and X is closed. After sorting this out in class with the teacher's help, José had a similar difficulty ten minutes later. José appeared not to have a behavioral schema that links situations like this to writing $X \in \mathcal{C}$, even though he knew formal conceptual things about sets. We suspect most mathematicians would not think students at this level would have this difficulty, but we think something like it is fairly common. We note that teaching students how features of sets are interrelated, whether formally or intuitively, is not the same as facilitating the formation of behavioral schemas useful in proving theorems, perhaps by providing additional relevant proving opportunities.

References

Hiebert, J., & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In J. Hiebert, (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 1-27). Hillsdale. NJ: Lawrence Erlbaum Associates.

James, W. (1890). *The principles of psychology*. New York: Holt.

Lau, H. C., Rogers, R. D., Haggard, P., & Passingham, R. E. (2004). Attention to intention. *Science*, 303, 1208-1210.

Libet, B., Gleason, C. A., Wright, E. W., & Pearl, D. K. (1983). Time of conscious intention to act in relation to onset of cerebral activity (readiness-potential): The unconscious initiation of a freely voluntary act. *Brain*, *106*, 623-642.

Margolis, H. (1993). *Paradigms and barriers: How habits of mind govern scientific beliefs*. Chicago: University of Chicago Press.

Mason, J., & Spence, M. (1999). Beyond mere knowledge of mathematics: The importance of knowing-to-act in the moment. *Educational Studies in Mathematics*, *28*, 135-161.

Norton, A., & D'Ambrosio, B. (May, 2008). ZPC and ZPD: Zones of teaching and learning. *Journal for Research in Mathematics Education.* To appear.

Pauen, M., Staudacher, A., & Walter, S. (Eds.). (2006). Special Issue on Epiphenomenalism. *Journal of Consciousness Studies*, *13*(1-2).

Pockett, S., Banks W. P., & Gallagher, S. (Eds.). (2006). *Does consciousness cause behavior?* Cambridge, MA: The MIT Press.

Selden, A., Selden, J., Hauk, S., & Mason, A. (2000). Why can't calculus students access their knowledge to solve non-routine problems? In A. H. Schoenfeld., J. Kaput, & E. Dubinsky, (Eds.), *Issues in mathematics education: Vol. 8. Research in collegiate mathematics education. IV* (pp. 128-153). Providence, RI: American Mathematical Society.

Selden, A., Selden, J., & McKee, K. (2008). The role of nonemotional cognitive feelings in constructing proofs. *Proceedings of the 11th Annual Conference on Research in Undergraduate Mathematics Education.* Available online.

Star, J. (2005). Reconceptualizing procedural knowledge. *Journal for Research in Mathematics Education*, 36, 404-411.

Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics, 12*, 151-169.

Tulving, E. (2000). Concepts of memory. In E. Tulving & F. I. M. Craik (Eds.), *The Oxford handbook of memory* (pp.33-43). Oxford: Oxford University Press.

Vergnaud, G. (1990). Epistemology and psychology in mathematics education. In P. Nesher & J. Kilpatrick (Eds.), *Mathematics and cognition: A research synthesis* *by the International Group for the Psychology of Mathematics Education* (pp. 14-30). Cambridge, UK: Cambridge University Press.

Weber, K. (2008). Learning advanced mathematical concepts by reading text. *Proceedings of the 11th Annual Conference on Research in Undergraduate Mathematics Education*. Available online.